New LEP constraints on some supersymmetric Yukawa interactions that violate R-parity

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ABSTRACT

We consider one-loop corrections to partial widths of the Z induced by supersymmetric Yukawa interactions that violate R-parity. The precise experimental values of the leptonic Z partial widths bound these Yukawa couplings, with the most interesting constraints being those on couplings involving the τ, since previous constraints on them were very mild.

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Of the many possible extensions of the Standard Model (SM), supersymmetry is considered to be one of the most promising candidates and, consequently, a significant amount of theoretical and experimental effort has been devoted to looking for signals of supersymmetry at present and future colliders. In particular, the minimal supersymmetric extension of the Standard Model (MSSM) [1] has been the subject of numerous investigations. In addition to the usual particles of the Standard Model, the MSSM contains their superpartners and two Higgs doublets. The gauge structure of the MSSM essentially replicates that of the Standard Model; there is no arbitrariness in the structure of the gauge interactions. This is, however, not true for the Yukawa sector of the MSSM. In addition to the usual Yukawa couplings of the fermions to the Higgs (responsible for the fermion masses), other interactions involving squarks or sleptons are possible.

The relevant part of the superpotential containing the Yukawa interactions involving squarks or sleptons in the MSSM is given in terms of the chiral superfields by

$$\mathcal{W}_{\not{R}} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \tag{1}$$

where the L_i and Q_i are $SU(2)$-doublet lepton and quark fields and the E_i^c, U_i^c, D_i^c are singlet superfields. In general, there are 45 such Yukawa couplings: 9 each of the λ- and λ''- types (because of the antisymmetry in the first (last) two generation indices of the former (latter) couplings) and 27 of the λ'-type. The couplings λ and λ' violate lepton (L) number, whereas the λ'' coupling violates baryon (B) number. The B- and L-violating couplings cannot be present simultaneously, because that would lead to very rapid proton decay. It is possible to forbid the existence of all three interactions in eq.(1) by imposing a discrete symmetry – R-parity. This discrete symmetry may be represented by $R = (-1)^{(3B+L+2S)}$, where S is the spin of the particle, so that the usual particles of the SM have $R = 1$, while their superpartners have $R = -1$.

The requirement that the MSSM Lagrangian be invariant under R-parity is sufficient to exclude each of the interactions in Eq.(1). There is, however, no compelling theoretical argument in favour of such a symmetry [2]. R-parity conservation is too strong a requirement to ensure proton stability – the latter can simply be ensured by assuming that either the L-violating or the B-violating couplings in Eq.(1) are present, but not both. Relaxing the requirement of R-parity conservation has important implications for supersymmetric particle searches at colliders : a superparticle can decay into standard particles *via* the R-parity violating couplings in eq.(1).

In this letter we examine the constraints on some of the L-violating λ'-type couplings imposed by precision LEP observables, particularly the $Z \to l^+ l^-$ partial decay widths. There are new triangle diagrams with Z, l^+ and l^- external lines involving λ'_{ijk} vertices with $i =$ lepton, $j =$ quark, $k =$ squark indices or $i =$ lepton, $j =$ squark, $k =$ quark indices. Since the magnitude of the new contribution depends on the mass of the fermion in the loop, only λ'_{i3k}-type couplings leading to internal top quark lines can

be constrained significantly by our considerations [1]. The constraints we derive require these couplings to be smaller than the $SU(2)$ gauge coupling or the top quark – Higgs Yukawa coupling if the squarks are ~ 100 GeV.

We should also recall that there exist important cosmological constraints [6] on R-parity-violating scenarios. Requiring that GUT-scale baryogenesis does not get washed out imposes $\lambda'' << 10^{-7}$ generically, though these bounds are model dependent and can be evaded [7]. Assuming $\lambda'' = 0$, the λ' couplings cannot wash out the initial baryon asymmetry by themselves. However, they can do so in association with a B-violating but $(B - L)$ conserving interaction, such as sphaleron-induced non-perturbative transitions. Since these processes conserve $\frac{1}{3}B - L_i$ for each lepton generation, the conservation of any one lepton generation number is sufficient [8] to retain the initial baryon asymetry. Therefore, the assumption that the smallest λ'-type coupling is less than $\sim 10^{-7}$ is enough to avoid any cosmological bound on the remaining λ'-type couplings. Hence, we proceed with a B-conserving but L-violating scenario, assuming $\lambda'' = 0$, some theoretical motivations for which can be found in ref. [9].

Since we are only interested in the interactions generated by the λ' couplings, we write this part of the Lagrangian out in terms of the component fields. In four-component Dirac notation, the complete set of Lagrangian terms with λ' couplings is given by

$$
\begin{aligned}
\mathcal{L} = \lambda'_{ijk}[&\tilde{\nu}^i_L \bar{d}^k_R d^j_L + \tilde{d}^j_L \bar{d}^k_R \nu^i_L + (\tilde{d}^k_R)^* (\bar{\nu}^i_L)^c d^j_L \\
-&\tilde{e}^i_L \bar{d}^k_R u^j_L - \tilde{u}^j_L \bar{d}^k_R e^i_L - (\tilde{d}^k_R)^* (\bar{e}^i_L)^c u^j_L] + \text{h.c.}
\end{aligned}
\tag{2}
$$

In our work, we concentrate on computing the one-loop corrections arising due to the interactions in Eq.(2) to the partial widths $Z \to l^+ l^-$ (where l^- and l^+ denote a charged lepton and the corresponding antilepton, respectively), because these widths are very precisely determined experimentally. Since the experimental determination of the leptonic width requires knowledge of the hadronic width of the Z, it is important also to know the effect of the λ'_{i3k} couplings on the hadronic channels, for example, $Z \to \bar{b}b$ [2] In fact, it is clear that the effect on Γ_b of the $i =$ lepton, $j =$ quark (squark), $k =$ squark (quark) coupling is negligibly small, since this vertex involves leptons in the loop, and hence gives corrections which are, at best, a factor m_τ^2/m_t^2 smaller than the corrections to the leptonic width. But a sizeable correction to Γ_b can arise due to the $i =$ slepton, $j =$ quark, $k =$ quark vertex, since the corresponding loops involve the top quark.

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[1]For a review of previous phenomenological constraints on λ'_{ijk}, see [3, 4], and for a recent complementary study, see [5].

[2]In fact, the dominant correction to the total hadronic width comes from $Z \to \bar{b}b(\bar{d}d, \bar{s}s)$ depending upon whether the R-violating Yukawa coupling is $\lambda'_{i33}(\lambda'_{i31}, \lambda'_{i32})$. We consider, of course, only one of them at a time. Moreover, since the tree-level SM predictions for $\Gamma_{b(d,s)}$ are same, the bounds on λ'_{i3k} are by and large the same irrespective of the choice of k for a common sparticle mass.

The tree level Z couplings to the left- and right-handed fermions are given by a_L^f and a_R^f, respectively, which appear in

$$M_\mu^{\text{tree}} = \frac{e}{s_W c_W} \bar{f}(p')\gamma_\mu(a_L^f P_L + a_R^f P_R)f(p). \qquad (3)$$

where

$$
\begin{aligned}
a_L^f &= t_3^f - Q_f s_W^2, \\
a_R^f &= -Q_f s_W^2.
\end{aligned}
\qquad (4)
$$

The Z couplings to the charge-conjugated fermions (f^c) are, therefore,

$$a_L^{f^c} = -a_R^f, \qquad\qquad a_R^{f^c} = -a_L^f. \qquad (5)$$

The triangle and self-energy diagrams that we need to compute for the correction to the processes $Z \to l^+l^-$ and $Z \to \bar{b}b$ are shown in Figs. 1a and 1b, respectively. We compute these diagrams in terms of the Passarino-Veltman B- and C-functions [10], corresponding to the two- and three-point integrals. For the purposes of presentation, we give explicit expressions for one diagram of Fig. 1a, $viz.$, the top-induced loop diagram contributing to $Z \to l^+l^-$. It is easy to see that only the coupling of the left-handed leptons to the Z is modified by this diagram, so that the amplitude due to the new contribution is

$$M_\mu^{(i)} = \frac{N_c}{16\pi^2}\frac{e\lambda'^2}{s_W c_W}\bar{l}(p')\gamma_\mu A_i l(p), \qquad (6)$$

where $i = 1, 2, 3$ and N_c is the colour factor. The A_i's are given by,

$$
\begin{aligned}
A_1 &= \left[a_L^{tc} m_t^2 C_0 - a_R^{tc}\{m_Z^2(C_{22} - C_{23}) + (d-2)C_{24}\}\right] P_L, \\
A_2 &= -2c_{\tilde{d}}\tilde{C}_{24}P_L, \\
A_3 &= a_L^l B_1 P_L.
\end{aligned}
\qquad (7)
$$

Here $A_{1,2}$ denote the contributions from the first and the second triangle diagrams, and the contributions of the two self-energy diagrams are jointly denoted by A_3. In A_2, we use $\tilde{C}_{24}$ to distinguish it from the C_{24} appearing in A_1, as the structures of the propagators for the two triangle diagrams are different. In the expression for A_2, $c_{\tilde{d}}$ refers to the coupling of the $\tilde{d}_R^k$-squark to the Z. We point out that the contributions from the individual diagrams are divergent, namely C_{24} in A_1, $\tilde{C}_{24}$ in A_2 and B_1 in A_3. But the divergence cancels when these amplitudes are added, and we are left with a finite correction. There is also a finite correction from the other set of triangle and self-energy diagrams in Fig. 1a, calculated analogously to eq.(7) with appropriate modifications. Together these finite parts make a new contribution to the partial width $Z \to l^+l^-$, which we denote by $\delta\Gamma_l$. We have evaluated the B- and C-functions required

using the code developed in ref.[11], cross-checking the results by using the standard Feynman parametrisation of the two- and three-point functions and then integrating them numerically.

To provide intuition into our numerical results, we also present analytic expressions valid in the limit $m_t, m_{\tilde{q}} \gg m_Z$, for an arbitrary value of the ratio $x \equiv m_t^2/m_{\tilde{q}}^2$. The sum of the A_i's in eq.(7) is given in this limit by

$$\sum_{j=1}^{3} A_j = \left[(a_L^{tc} - a_R^{tc})\eta_2(x) + \frac{m_Z^2}{3m_t^2} \left\{ a_R^{tc}\eta_1(x) + c_{\tilde{d}}\eta_3(x) \right\} \right], \tag{8}$$

where η_1, η_2 and η_3 are given by

$$\begin{aligned}
\eta_1(x) &= \frac{-11x + 18x^2 - 9x^3 + 2x^4}{6(1-x)^4} - \frac{x \ln x}{(1-x)^4} \simeq 0 \ (x \to 0), \\
\eta_2(x) &= -\frac{x}{1-x} - \frac{x \ln x}{(1-x)^2} \simeq 0 \ (x \to 0), \\
\eta_3(x) &= \frac{2x - 9x^2 + 18x^3 - 11x^4}{6(1-x)^4} + \frac{x^4 \ln x}{(1-x)^4} \simeq 0 \ (x \to 0).
\end{aligned} \tag{9}$$

Eqs (8) and (9) exhibit the expected decoupling in the limit of large $m_{\tilde{q}}$, and match qualitatively the exact numerical results presented later even for smaller values of $m_{\tilde{q}}$.

We neglect left-right squark mixing, which is expected to be significant only for the $\tilde{t}$ [12]. As noted earlier, the diagram that contributes dominantly to $\delta\Gamma_l$ is that with the top quark in the loop, and this diagram contains a b-squark, which justifies our neglect of left-right squark mixing. Similarly, from a computation of the diagrams in Fig. 1b, we obtain a modification to Γ_b, which we denote by $\delta\Gamma_b$ [3]. We observe that the quantity $\delta\Gamma_l$ is a function of the squark mass, whereas $\delta\Gamma_b$ depends *mainly* on the slepton mass. Although the squark and slepton masses are completely free parameters phenomenologically, the usual assumption of unification at some Grand Unified scale would normally predict the physical slepton masses to be smaller than the squark masses: the low-energy splitting can be calculated exactly by renormalization group evolutions from some initial conditions in specific models. For our purposes, it is a safe approximation to work with the slepton mass equal to half of the squark mass: our results are insensitive to plausible variations from this approximation.

We compute numerically the corrections to the ratio R_l, defined as:

$$R_l = \frac{\Gamma_h}{\Gamma_l} \quad (l = e, \mu, \tau). \tag{10}$$

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[3]It may be noted that, for λ'_{i33} couplings, another set of triangle and self energy diagrams with external b_L- and internal b_R-lines in addition to the diagrams shown in Fig. 1b has to be taken into account. As has been pointed out, $Z \to \bar{b}b$ also receives contributions from internal lepton and squark lines. However, these contributions are small and are neglected.

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Defining the quantities Δ_f $(f = l, b)$ by

$$\Delta_f = \frac{\Gamma_f - \Gamma_f^{\text{SM}}}{\Gamma_f^{\text{SM}}}, \tag{11}$$

one has the following expression for the change in R_l:

$$\delta R_l \equiv R_l - R_l^{\text{SM}} \approx R_l^{\text{SM}} R_b^{\text{SM}} \Delta_b - R_l^{\text{SM}} \Delta_l. \tag{12}$$

Using this equation and, since δR_l always turns out negative, comparing the 2-σ experimental upper bound on R_l with the corresponding SM prediction, we obtain the maximum allowed value of λ'_{i3k} as a function of the squark mass. We use the following experimental values [13]: $R_e = 20.850 \pm 0.067$, $R_\mu = 20.824 \pm 0.059$, $R_\tau = 20.749 \pm 0.070$. The SM predictions for R_l and R_b are 20.786 and 0.2158 respectively for the following choice of input parameters: $m_t = 175$ GeV, $m_H = 300$ GeV and $\alpha_s = 0.126$ [13]. The ratio R_l is quite insensitive to these values, so our results do not change much by varying these parameters. Fig. 2 shows the values of λ'_{max} obtained as functions of the squark mass for e, μ and τ final states. We observe that for $m_{\tilde{q}} = 100$ GeV, $\lambda'_{33k} \leq 0.45$, $\lambda'_{23k} \leq 0.56$ and $\lambda'_{13k} \leq 0.63$ at the two-standard deviation level. We observe that the previous low energy bound on the coupling λ'_{133} is much stronger [4] than the one obtained from our analysis, while the rest of our bounds are new, with the exception of those on λ'_{131} and λ'_{231}. Our bounds could be significantly improved as more data accumulate on the Z-peak. A compilation of our and previous phenomenological bounds on λ'_{i3k} are displayed in Table 1, where, for the sake of consistent comparison with the previous bounds, we also present the constraints we obtain at the one-standard deviation level.

In passing, we note that the above interactions may also affect the SM predictions of the forward-backward charge asymmetries (A_{FB}^l), the τ-polarisation asymmetry (A^τ) or the left-right asymmetry (A_{LR}), however, in view of our constraints, the effects are below the experimental sensitivity, as can be inferred from the similarity of the λ'-type R-parity-violating interactions to leptoquark-induced contributions to the same processes [15].

Our bounds have implications for sparticle decays in models with R-violation. In the past [16], sparticle decays into leptons, particularly τ and/or ν_τ, have been discussed from a phenomenological point of view. Our bounds imply that decays *via* λ'-type couplings are unlikely to dominate over R-parity conserving decays of heavier sparticles into gauginos, at least if sparticles weigh ~ 100 GeV. Searches for experimental signatures of R-parity violation should therefore focus on decays of the lightest supersymmetric particle, which could still be rapid enough to occur inside a detector.

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[4] We wish to point out that the upper limit of ν_e Majorana mass can be used to put constraints on λ'_{1jk} *only when* $j = k$, and not in the general case for any j and k as suggested in ref.[14].

Previous analyses				Present analysis					
λ'_{13k}	Limits	λ'_{23k}	Limits	λ'_{13k}	Limits	λ'_{23k}	Limits	λ'_{33k}	Limits
131	0.26	231	0.22*	131	0.63	231	0.56	331	0.45
					(0.51)		(0.44)		(0.26)
132		232		132	0.63	232	0.56	332	0.45
					(0.51)		(0.44)		(0.26)
133	0.001	233		133	0.63	233	0.56	333	0.45
					(0.51)		(0.44)		(0.26)

Table 1: Limits on the λ'_{i3k} couplings from the previous and present analyses for $m_{\tilde{q}} = 100$ GeV. The number marked by ($*$) in the column of previous analyses corresponds to a 2-σ limit, while the other numbers correspond to 1-σ limits (see [4, 14]). In the columns of the present analysis, we give numbers corresponding to 2-σ limits followed by 1-σ limits in brackets.

In conclusion: we have considered the effect of the R-parity-violating couplings λ' on the Z partial widths. By studying the effect of the interactions induced by these couplings on the ratio of the hadronic and leptonic widths, we have been able to place reasonably stringent constraints on the couplings λ' as functions of the masses of the squarks and sleptons. The available data allow us to get interesting bounds which are complementary to those previously existing, particularly for interactions involving the τ lepton. Our bounds could be improved significantly with more precise data.

Acknowledgements

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[16] See, for example, Ellis et al in [2].
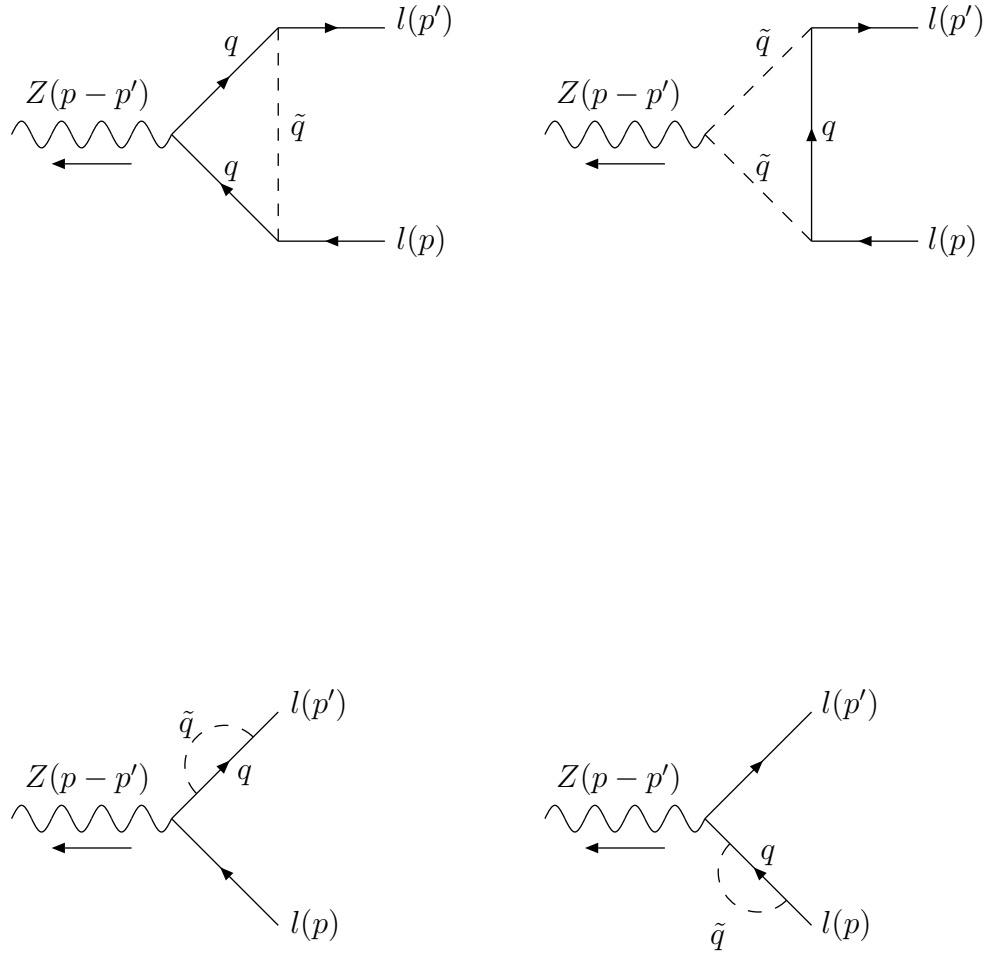
$$q = t^c, b; \ \tilde{q} = \tilde{b}, \tilde{t}^*$$

Figure 1a: The one-loop Feynman diagrams contributing to the $Z \to l^+l^-$ vertex correction due to the R-parity-violating coupling λ'.
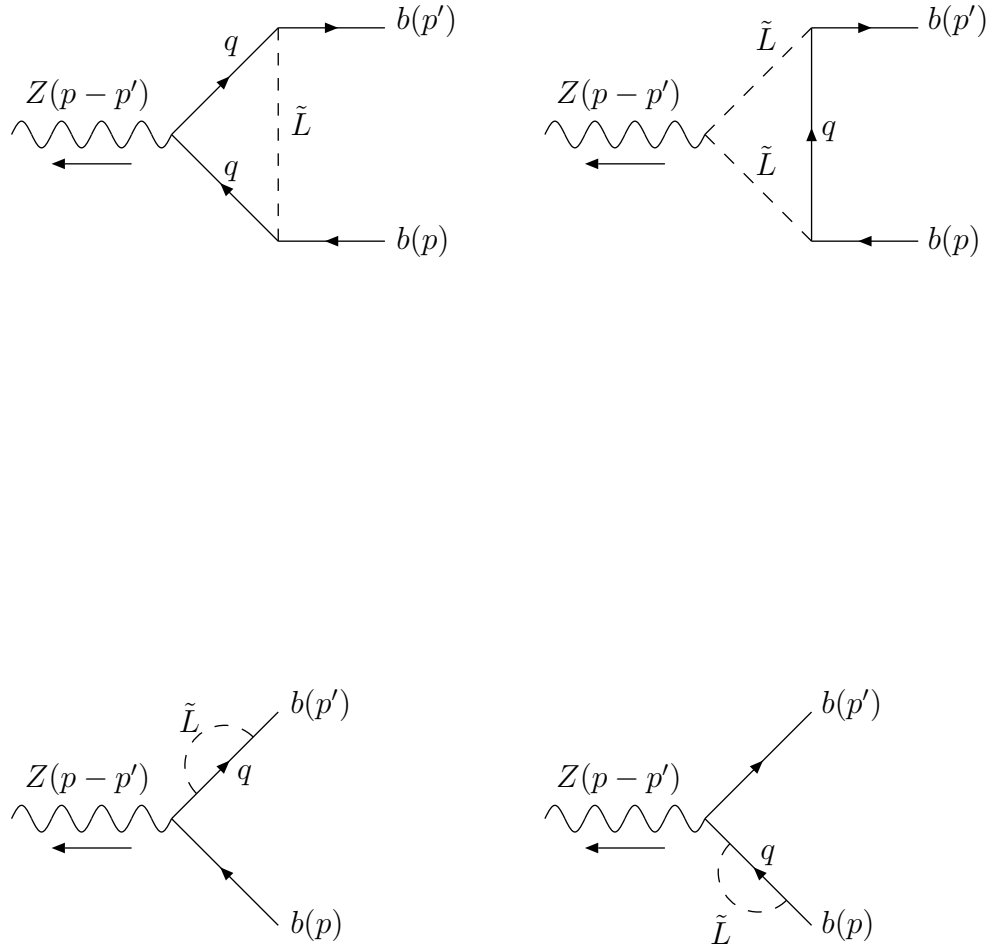
$$q = t, b; \ \tilde{L} = \tilde{l}, \tilde{\nu}_l$$

Figure 1b: The one-loop Feynman diagrams contributing to the $Z \to \bar{b}b$ vertex correction due to the R-parity-violating coupling λ'.
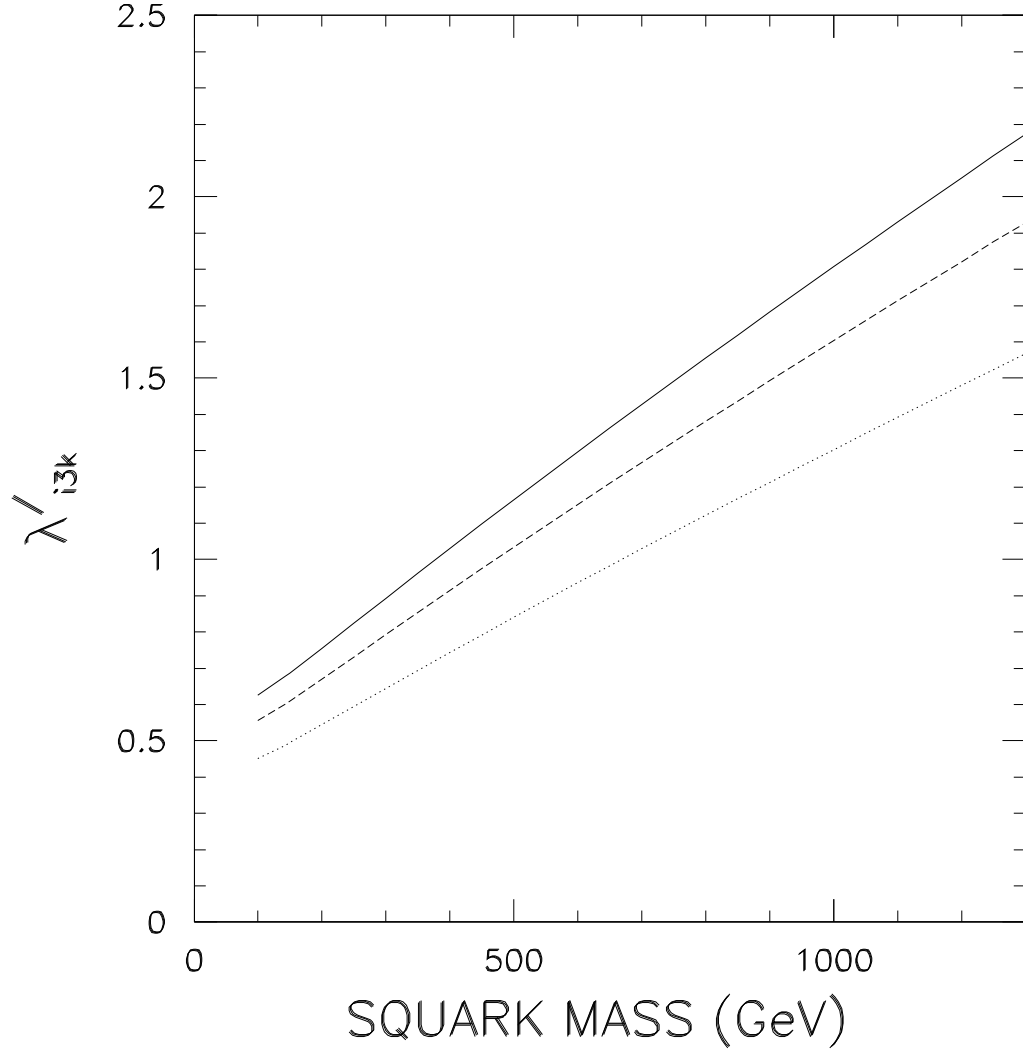
Figure 2: The maximum values of the λ'_{i3k} (2-σ limits) as functions of the squark mass allowed by the e, μ and τ partial widths of the Z (solid, dashed and dotted lines, corresponding to $i = 1, 2$ and 3, respectively).